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Track 7. Intelligent Robots & Systems

Track7 – S1. Robotics I

Chairs: Antoni Grau and Yolanda González

Sala TV – 11:30-13:00 – Monday, September 13rd, 2010

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2	MF-002607	Sampling-based Safe Path Planning for Robotic Manipulators <i>Bakir Lacevic, Paolo Rocco</i>
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Chairs: Antoni Burguera and Fabrizio Abrate

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2	MF-002291	Beyond RatSLAM: Improvements to a Biologically Inspired SLAM System <i>Niko Sünderhauf, Peter Protzel</i>
3	MF-002275	Cooperative Robotic Teams for Supervision and Management of Large Logistic Spaces: Methodology and Applications <i>Fabrizio Abrate, Basilio Bona, Luca Carlone, Marina Indri</i>
4	MF-000248	Multi-source Sound Localization using the Competitive K-means Clustering <i>Byoung-gi Lee, JongSuk Choi</i>
5	MF-000922	Comparison of Mapping Techniques in Appearance-Based Topological Maps Creation <i>Lorenzo Fernandez Rojo, Luis Paya Castello, Oscar Reinoso Garcia, Jose Maria Marin Lopez, Arturo Gil Aparicio</i>

Comparison of Mapping Techniques in Appearance-Based Topological Maps Creation.

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Abstract

In this paper we compare two methods to carry out topological mapping using only visual information captured by a robot. This map should contain enough information so that the robot can estimate its position and orientation and redundant information should be removed to get an acceptable computational cost during the localization process. Apart from this, it is also important to know the topology of the map created since it will make possible a high-level planification of the path to move to the target points. We propose to build this topological map only using the panoramic images taken by an omnidirectional vision system and using appearance-based methods. We have carried out an exhaustive experimentation to study the validity of the proposed methods and to perform an objective comparison between them. Also, we have tested the processing time to create the topological map.

1. Introduction

In most cases, when a mobile robot or a team of mobile robots have to carry out a task in a large environment, they have to take decisions about their localization and about the trajectory to follow to move from their current position to the target point. A map or an internal representation of the environment is needed to solve this problem. Omnidirectional visual systems are a widespread kind of sensors used with this aim due to their low cost and the large amount of information they provide. The study of algorithms to construct a representation of the environment from visual information is a major area of research in robotics. One of the existing options consists in using appearance-based techniques, which work with the information in the images as a whole. As an example, [4] describes a probabilistic SLAM (Simultaneous Localization and Mapping) approach to the problem of recognizing places based on their appearance and in [3] a probabilistic bail-out condition test to accelerate an appearance-only SLAM system is applied. These techniques offer a systematic and intuitive way to construct the map.

Nevertheless, as no relevant information is extracted from the images, it is usually necessary to apply some compression technique to reduce the computational cost of the mapping and localization processes.

A widely used method that meets this requirement is PCA (Principal Components Analysis). For example, [8] creates a database by means of PCA. But the main problem of PCA methods is that they are not inherently invariant to the orientation of the robot. [13] studies the problem of invariance to ground-plane rotations taking into account this fact and [7] applies these concepts to build an appearance-based map of an environment including information not only about the localization where the images were taken, but also about the possible orientations at that points. Other researchers make use of Fourier transform methods to get the most relevant information from the images. As an example, [9] defines the concept of Fourier Signature and [11] applies this concept to the construction of appearance-based dense maps. The Fourier Signature exploits better the rotational ground-plane invariance and concentrates the most relevant information in the low frequency components of each row of the panoramic image.

The appearance of an image depends strongly of the lighting conditions of the environment to map [1]. The images can be previously treated to minimize these effects, as [5] does by means of applying a bank of homomorphic filters.

With respect to the mapping problem, the current research can be classified into two approaches: *metric* and *topological*. The first one consists in modelling the environment using a metric map obtained with geometrical accuracy when representing the position of the robot in the map. For example [10] describes a sonar-based mapping system developed for mobile robot navigation and [2] analyses the performance of several established mapping techniques using identical test data. On the other side, topological maps are graphical models of the environment that capture places and their connectivity in a compact form. An example of this method is presented in [9] where a topological map of the environment is obtained by means of the application of a method based on the physics of harmonic oscillators. Also [15] presents a method for topological SLAM using fast vision techniques. [14] studies how to build a topological map of large indoor and outdoor

environments using local features extracted from omnidirectional images and the epipolar constraint, and a clustering method to perform localization more efficiently. [12] presents an appearance-based method for path-based map learning by means of a clustering of the PCA features extracted from a set of panoramic images into distinctive visual aspects.

In this paper a methodology to build a robust appearance-based topological map under changing lighting conditions is presented. We show how it is possible to extend this method to construct the topological map incrementally. The main advantage of our incremental method comparing to non-incremental methods is the possibility of constructing the map in real time while the robot is exploring the environment. Also, as we show throughout the paper, it avoids possible errors in the construction of the map when a incremental method is used.

The work is structured as follows. Section 2 makes a brief review of homomorphic filtering to remove the dependence on changes in the lighting conditions and a method for non-incremental topological mapping. Section 3 presents a technique for building topological maps incrementally. Section 4 shows the results of the experiments. Finally, the conclusions of the work are presented in section 5.

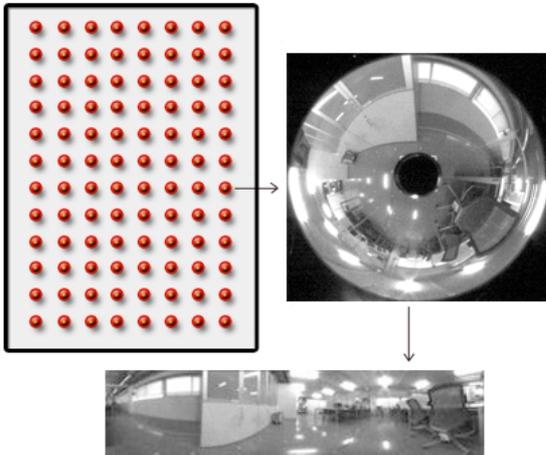


Figure 1. 40x40 cm grid, omnidirectional image and panoramic image taken in a laboratory.

2. Topological mapping

To construct the topological map, a graph model in which nodes represent positions and links represent the connectivity between them has been used. To build the maps we have captured several omnidirectional image sets in several predefined grids with different shapes and number of images, both in unstructured indoor environments (a laboratory, a room, etc.) and in a

structured indoor environment (a corridor). We can see an example on Fig. 1.

On the other hand, as appearance-based methods do not extract any relevant point or region from the images, but use them as a whole, some method that allow us to reduce the high computational cost further reducing the amount of memory necessary for the mapping and localization processes must be used. Among the different compression techniques applied we have chosen the Fourier Signature, due to the fact that we can achieve a high compression level, it is a very fast method and it presents invariance to changes in the orientation of the panoramic images [11]. It is also an incremental method because we do not need information from the rest of the images to compute the Fourier transform (what differs from PCA).

Although the goal in this work is to build a topological map from a database consisting of the Fourier Signature of the different images, it is necessary to put into relief that the map created, and therefore the database must be robust against small variations in the environment, such as the change of the state or position of some objects and against lighting conditions. There are different methodologies to provide robustness to the map created. [5] shows how it is possible to increase the accuracy in locating a robot in a previously created map applying Homomorphic filtering techniques on the panoramic images captured. This is the reason why in this paper to provide robustness to the map, panoramic images are previously filtered employing a Homomorphic filter.

2.1. Homomorphic filter

By means of the application of a Homomorphic Filtering we can filter separately the luminance and reflectance components of an image [6]. It allows us to control the influence of each component on the image appearance. To separate the components of luminance and reflectance, the Homomorphic Filter uses the Neperian logarithm operator on the image:

$$\begin{aligned} f(x,y) &= i(x,y) \times r(x,y) \\ z(x,y) &= \ln(f(x,y)) \\ z(x,y) &= \ln(i(x,y)) + \ln(r(x,y)) \end{aligned} \quad (1)$$

Where $f(x,y)$ corresponds to the panoramic image that can be expressed as a multiplication of the luminance $i(x,y)$ and the reflectance $r(x,y)$ components. Once the components have been separated, the image can be filtered in the frequency domain applying previously the 2D Discrete Fourier Transform:

$$\begin{aligned} \mathfrak{F}(z(x,y)) &= \mathfrak{F}(\ln(i(x,y))) + \mathfrak{F}(\ln(r(x,y))) \\ \mathfrak{F}(z'(x,y)) &= \mathfrak{F}(z(x,y)) \cdot H(u,v) \end{aligned} \quad (2)$$

Where $H(u,v)$ is the filter transfer function in the frequency domain. As low frequency components are associated with the illumination of the image and the high frequency ones with the reflectance, we apply a high pass filter constructed from a Butterworth low pass filter, to reduce the effects of changes in the lighting of the scenes [6]:

$$\begin{aligned}
 D(u,v) &= (u^2 + v^2)^{1/2} \\
 H_{lp}(u,v) &= \frac{1}{1 + \left[\frac{D(u,v)}{D_0} \right]^{2n}} \\
 H'_{hp}(u,v) &= 1 - H_{lp}(u,v) \\
 H_{hp}(u,v) &= (\alpha_h - \alpha_l) \cdot H'_{hp}(u,v) + \alpha_l
 \end{aligned} \tag{3}$$

Where $D(u,v)$ is the distance to the origin in the frequency domain. D_0 is the filter cut-off frequency to construct the low pass filter, n is the order of the filter and $H_{lp}(u,v)$ is the low pass filter transfer function in the frequency domain. The last two expressions are used to build the high pass filter from the low pass filter, where α_h and α_l correspond to the maximum and minimum value of Homomorphic filter and $H_{hp}(u,v)$ is the high pass filter transfer function in the frequency domain. Fig. 2 shows an example of Homomorphic Filter Transfer Function. The optimal parameters are the choice of a D_0 of 50Hz, n equal to 3, α_h equal to 0.21 and α_l equal to 0.20. Further information can be found in [5].

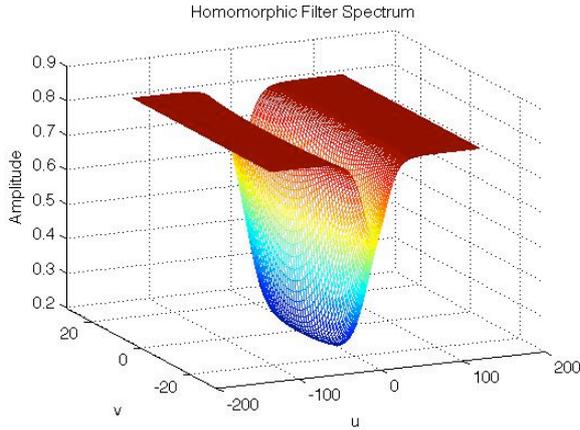


Figure 2. Transfer Function Amplitude of a Homomorphic Filter.

2.2. Mass-spring-damper model

The topological map is a graph whose nodes correspond to distinct locations in the environment and whose edges model the neighboring relations between the nodes. To create the map, first we have captured the image grid by teleoperating the robot through the

environment. Once all the images have been captured, we have implemented a method that allows us to create a topological map from them, without taking into account the storage order. The method is capable of ordering the captures and situates them in the corresponding place of the topological map.

To do this a method based on Hooke's law and Newton's second law, known as mass-spring-damper system [9] has been used. Fig. 3 shows an example of the physical principle to build the map. Each particle P_i is an image and the springs S_{ij} connecting the particles together represent the distances between images captured. Each particle of the system will be connected with other particles (with all of the nearest). To calculate the lengths of the springs of the system we use the Euclidean Distance between the Fourier Signature of stored images. When we let the system to evolve freely, it is expected to tend to a similar topology compared to the real system [9].

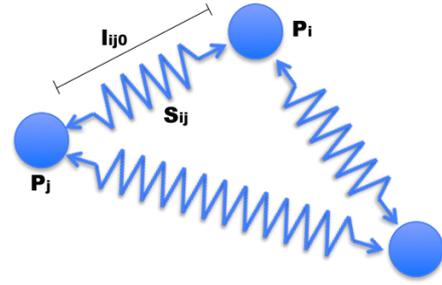


Figure 3. Spring model for topological map.

To construct the mass-spring-damper system we have taken into account the following system of forces:

$$\begin{aligned}
 \vec{F}_i &= \sum_{S_{ij} \in S} (-k_{ij} \cdot (l_{ij0} - l_{ij}) - \kappa_{ij} \cdot (v_i - v_j)) \\
 \vec{a}_i &= \vec{F}_i / m_i \rightarrow \dot{\vec{v}}_i = \vec{a}_i \rightarrow \dot{\vec{r}}_i = \vec{v}_i \\
 \vec{r}_i(t + \Delta T) &= \vec{r}_i(t) + \vec{v}_i(t) \cdot \Delta T + 1/2 \cdot \vec{a}_i(t) \cdot \Delta T^2 \\
 \vec{v}_i(t + \Delta T) &= \vec{v}_i(t) + \vec{a}_i(t) \cdot \Delta T
 \end{aligned} \tag{4}$$

Where the first equation corresponds to Hooke's Harmonic Oscillator Law and depends both on the length of the spring l_{ij} and on the difference of velocities where k_{ij} is the elastic and κ_{ij} the damping constant. The second expression refers to Newton's second law and allows to obtain the equations of motion of the system from the force \vec{F}_i provided by the Hooke's law and the mass m_i of the particle. To simplify the system we have used the same mass for all particles. This mass is equal to 1. Finally the last two equations show how to calculate the position \vec{r}_i and velocity \vec{v}_i of every particle of the system at each iteration.

It is interesting to highlight the importance of the parameter ΔT in the system relaxation time. If ΔT takes a too large value, the system is very unstable and

therefore it will be difficult to reach the steady-state. By contrast, if a too small value is chosen, it will take too long to reach the rest. This is the reason why we have achieved a compromise between speed and stability. To do this we set a maximum number of steps s_{tot} and we let ΔT be dependent on it:

$$\Delta T = \xi \cdot \left(1 - \frac{s}{s_{tot}}\right) \quad (5)$$

Where s corresponds to the number of steps until a given time and ξ is a constant.

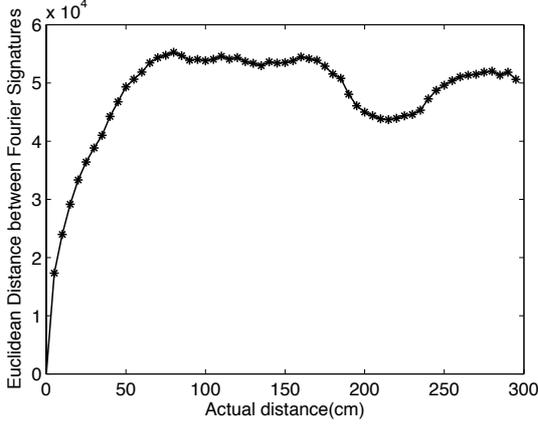


Figure 4. Euclidean distance between Fourier Signatures versus real geometric distance between the points where images were captured.

Another issue to consider is the value of the elastic constant k_{ij} of each spring system. As it can be observed in Fig. 4, the Euclidean distance between the Fourier projections behaves approximately linear in the surroundings of the point where the image was captured, but this linearity disappears as we move away from it. To solve the problem we have taken elastic constants dependent on Euclidean distance L_2^{ij} between the Fourier projections and, to build the map, we only use the images closer to each image. So we just add springs between the images that are below a distance threshold.

3. Incremental topological mapping

One of the main problems that it can be found when constructing a topological map from the stored database resides in the fact that the computational cost increases exponentially with the number of images captured, due to the growing number of forces on each particle of the mass-spring-damper system. Also it is necessary to have all the images stored in the database to create the topological map, so it is only possible to build it in an offline process, once all the images have been captured.

To avoid this problem, it is interesting to exploit the benefits offered by working with the Fourier Signature.

One of the most important features is the fact that the Fourier Signature is an inherently incremental method. Thus we can build the topological map incrementally and also in real time.

3.1. Incremental mass-spring-damper model

The incremental topological map has been implemented from a process based on the system used in the previous section to construct the non-incremental topological map. To build the map incrementally we have several options. When a new image arrives, we can relax only the new particle with respect to other particles that are already in the system or otherwise we can relax the whole system of particles. After several experiments we reached the conclusion that the best option was a compromise between both of them. Every time we add a new particle to the system we allow this particle to relax while maintaining the position of other particles fixed. Once the particle is at rest, the whole system relaxes. So, the system compensates the nonlinearity that occurs in the Euclidean distance as we move away from the particle metrically.

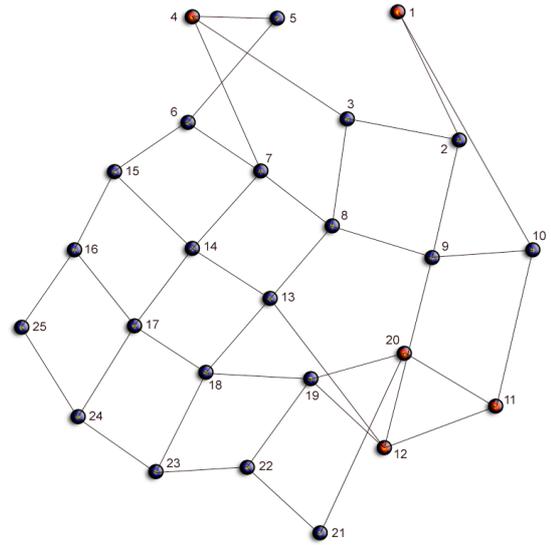


Figure 5. False minimum in a square grid of 5x5 images for non-incremental topological mapping.

Besides providing the benefits mentioned above, when the topological map is built incrementally, a very important advantage is provided to the system, it is more stable and does not produce false minima in the relaxation of the system. In Fig. 5 an example of a false minimum in a square grid of 5x5 images for non-incremental topological mapping can be seen. In structured environments where visual appearance of far points may be similar, false minima are a common problem.

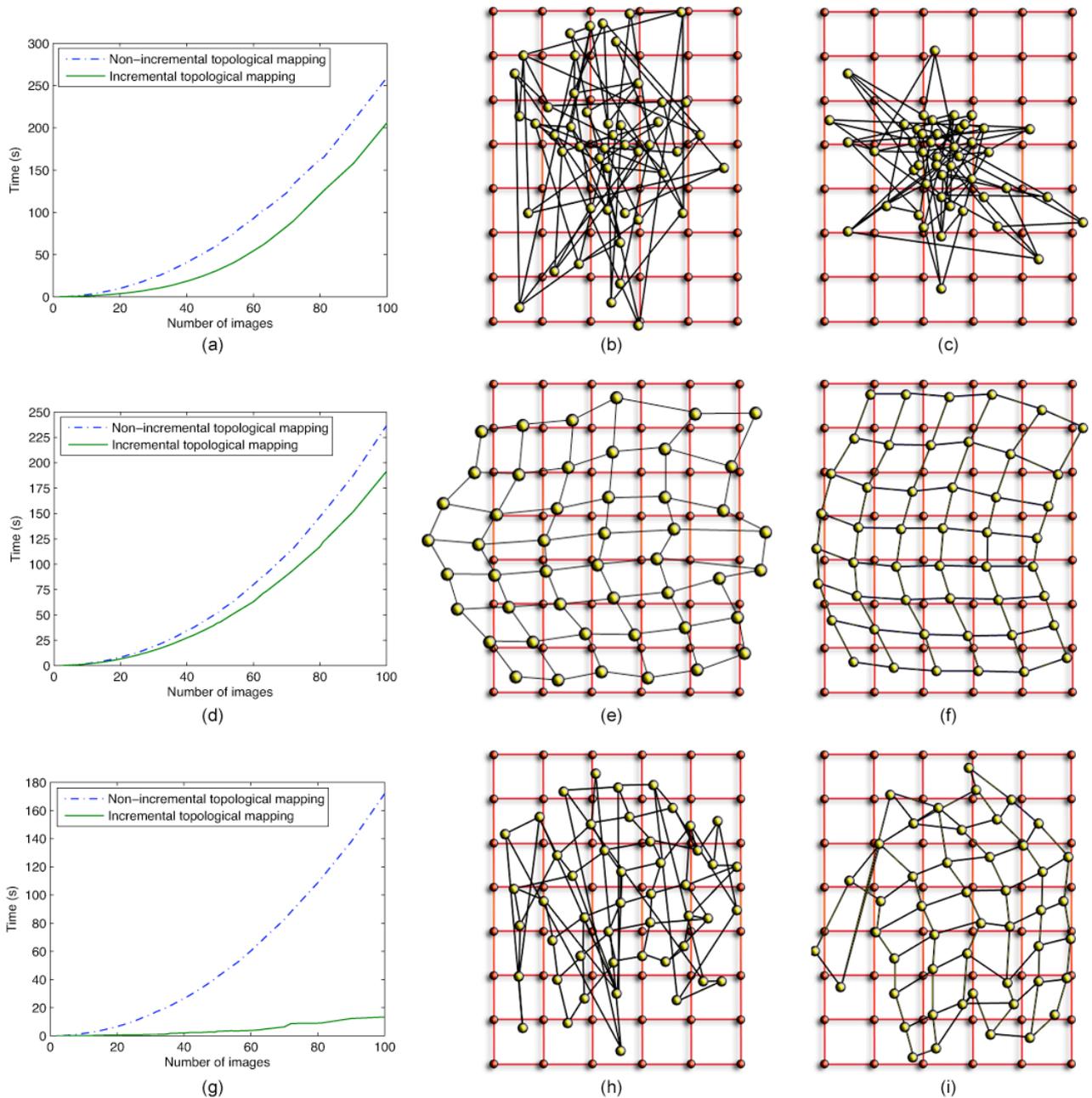


Figure 6. (a) Time elapsed to build a non-incremental topological map (discontinuous line) and an incremental topological map (continuous line), (b) comparison between the real map (small red circles) and the topological map obtained by the non-incremental method (big yellow circles), (c) comparison between the real topological map (small red circles) and the topological map obtained by the incremental method (big yellow circles), for ξ equal 0.5. (d), (e) and (f) show the same results for ξ equal 0.1 and (g), (h) and (i) for ξ equal 0.01.

4. Experimental results

The results obtained from experiments are presented in this section. Several sets of omnidirectional images with different topologies, varying the shape, the number

of images and the distance between them, have been taken to perform the experiments. Once all the scenes have been obtained, these have been transformed to panoramic images and we have obtained the corresponding Fourier signature. We have a total of 6 sets, whose main features are shown on table 1.

Set	Size x (Images)	Size y (Images)	Resolution	Grid step
Lab. 1	10	15	56x256 pixels	30 cm
Lab. 2	10	20	56x256 pixels	50 cm
Office	6	8	56x256 pixels	50 cm
Hall	12	9	56x256 pixels	10 cm
Corr. 1	22	10	56x256 pixels	10 cm
Corr. 2	35	10	56x256 pixels	10 cm

Table 1. Relevant physical parameters of the image sets.

The Euclidean distance L_2 between the main harmonics in the Fourier Signature of each image has been used to calculate the distance for each spring of the mass-spring-damper system. We work only with the first 16 harmonics of each row of Fourier Signature because, as [11] shows, it is desirable to reject the harmonics in the upper spectrum of the Fourier signature.

Time consumption is used as a parameter to compare the two methods of topological mapping. Moreover, to check the dependence of both methods regarding the time to build the map and the accuracy of the resulting map, the time constant ξ has been used. Taking it into account, we have compared different topologies obtained for each grid of images using both methods.

When we construct the topological map using the non-incremental method, as it can be observed on Fig. 6 (a), (d) and (g), the computing time increases exponentially with the number of images. In this case, the time for each iteration has a mean of 2.51 sec and a standard deviation of 0.12 sec for ξ equal 0.5, 2.32 sec of mean and 0.10 sec of standard deviation for ξ equal 0.1, and 1.74 sec of mean and 0.11 sec of standard deviation for ξ equal 0.01. The computing time increases exponentially with the number of images because it increases the number of neighbors for each image and therefore, at each iteration, a greater number of forces in the mass-spring-damper system must be computed. On the other hand when the incremental method it is used, it can be observed that the time grows to a lesser extent. In this case, the time for each iteration has a mean of 2.01 sec and a standard deviation of 0.11 sec for ξ equal 0.5, 1.91 sec of mean and 0.09 sec of standard deviation for ξ equal 0.1, and 0.16 sec of mean and 0.01 sec of standard deviation for ξ equal 0.01. As we increase ξ , the computation times increase, but when we use too high or too low values for ξ , the shape obtained does not represent the real shape of the map (Fig. 6 (b), (c), (e), (f), (h) and (i)). That is why a compromise between the computation times versus the accuracy of the shape obtained comparing to the real shape must be attained.

As we can see on Fig. 6 (e) and Fig. 6 (f), with the incremental method it can be obtained a topological map

that represents the shape of the real map of the environment by tuning correctly the value of ξ . Fig. 6 (e) shows the topological map obtained by applying non-incremental topological mapping and Fig. 6 (f) shows the topological map obtained by applying incremental topological mapping. The Incremental approach, besides improving the computing time, allows us to build a topological map that approximates better the topology of the real map of the environment. Finally, we must also take into consideration that during the course of the experiments, when we used the non-incremental method often false minimum appeared. However, by building the topological map incrementally a correct topological map was always obtained.

It should be stressed that, although the results shown in Fig. 6 correspond only with the results for the *Office Set* (table 1), for the process of tuning of the time constant ξ , we have used all sets of images that appear on table 1. It is why we can assert that our incremental topological mapping improves the results obtained regarding the non-incremental method.

5. Conclusions and future work

This paper shows how it is possible to construct a topological map of the environment incrementally from a set of omnidirectional images (views) obtained on a grid within the environment using only the appearance of each omnidirectional image without extracting salient points or regions.

The database has been built by applying a compression process to the visual information. We have previously applied a bank of Homomorphic filters on the panoramic images captured to obtain a robust map against illumination changes in the environment. Also, we have used the Fourier signature due to the fact that it presents a good performance in terms of amount of memory and computation time needed to build the database and it is also invariant to ground-plane rotations and an inherently incremental method, to compress the information.

For the creation of a robust topological map of the real environment we have presented two methods. As we have shown, the application of the non-incremental method (topological mapping) allow us to obtain a topological map of the environment that in most cases corresponds roughly with the actual shape of it, but sometimes erroneous topologies are presented. However, applying an incremental topological mapping allow to obtain a topological map of the environment that in all cases corresponds roughly with the actual shape and also reduces the computation time compared to the previous method. The incremental method also permits building the map online, while the robot is exploring the environment.

This work opens the door to new applications of appearance-based methods in mobile robotics. As we

have shown, the topological map created is robust against changes of lighting conditions, and permits us to know the actual topology of the map. With our incremental method, the computing time and the shape of the topological map obtained are improved. Moreover thanks to our method, it is possible to build the map online. If we know the topology of the environment and we know which node is the robot situated in, we will be able to create an algorithm that allows the robot to reach the objective points travelling the shortest path.

Further plans for future work include a study of the performance of our method when occlusions occur in the environment. In addition, a study of the possibility of using appearance-based Monte Carlo localization and mapping methods as well as appearance-based SLAM methods, will be carried out.

6. Acknowledgements

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References

- [1] Y. Adini, I. Moses, S. Ullman, 1997. Face recognition: the problem of compensating for changes in illumination direction. In *IEEE Trans. Pattern Analysis and Machine Intelligence*, Vol. 19, No. 7, pp. 721-732.
- [2] T. Collins, J.J. Collins, C. Ryan, 2007. Occupancy Grid Mapping: An Empirical Evaluation. In *Proceedings of the 15th Mediterranean Conference on Control & Automation*.
- [3] M. Cummins, P. Newman, 2008. Accelerated Appearance-Only SLAM. In *Proc. IEEE International Conference on Robotics and Automation (ICRA'08)*.
- [4] M. Cummins, P. Newman, 2008. FAB-MAP: Probabilistic Localization and Mapping in the Space of Appearance. In *The International Journal of Robotics Research*, Vol. 27, No. 6, pp. 647-665.
- [5] L. Fernández, L. Payá, O. Reinoso, A. Gil, M. Juliá, 2010. Robust methods for robot localization under changing illumination conditions. Comparison of different filtering techniques. In *Int. Conf. on Agents and Artificial Intelligence*, Vol. 2, pp. 223-228.
- [6] R.C. Gonzalez, R.E. Woods, 1993. In *Digital Image Processing. Ed. Addison Wesley*.
- [7] M. Jogan, A. Leonardis, 2000. Robust Localization Using Eigenspace of Spinning-Images. In *Proc. IEEE Workshop on Omnidirectional Vision*, Hilton Head Island, USA, pp. 37-44, IEEE.
- [8] B. Kröse, R. Bunschoten, S. Hagen, B. Terwijn, N. Vlassis, 2004. Household robots: Look and learn. In *IEEE Robotics & Automation magazine*. Vol. 11, No. 4, pp. 45-52.
- [9] E. Menegatti, T. Maeda, H. Ishiguro, 2004. Image-based memory for robot navigation using properties of omnidirectional images. In *Robotics and Autonomous Systems*. Vol. 47, No. 4, pp. 251-276.
- [10] H. Moravec, A. Elfes, 1985. High Resolution Maps from Wide Angle Sonar. In *Proceedings of the 1985 IEEE International Conference on Robotics and Automation*, pp. 116-121.
- [11] L. Payá, L. Fernández, O. Reinoso, A. Gil, D. Úbeda, 2009. Appearance-based dense maps creation. Comparison of compression techniques with panoramic images. In *Int. Conf. on Informatic in Control, Automation and Robotic*, Vol. 1, pp. 238-246.
- [12] A. Stimec, M. Jogan, A. Leonardis, 2008. Unsupervised learning of a hierarchy of topological maps using omnidirectional images. In *International Journal of Pattern Recognition and Artificial Intelligence (IJPRAI)*. Vol. 22, No. 4, pp. 639-665.
- [13] M. Ueonara, T. Kanade, 1998. Optimal approximation of uniformly rotated images: relationship between Karhunen-Loeve expansion and Discrete Cosine Transform. In *IEEE Transactions on Image Processing*. Vol. 7, No. 1, pp. 116-119.
- [14] C. Valgren, A. Lilienthal, 2008. Incremental Spectral Clustering and Seasons: Appearance-Based Localization in Outdoor Environments. In *Proc. IEEE Int. Conf. on Robotics and Automation*, pp. 1856-1861.
- [15] F. Werner, F. Maire, J. Sitte, 2009. Topological SLAM using fast vision techniques. In *Federation of International Robot-soccer Association*, LNCS 5744, pp. 187-19.