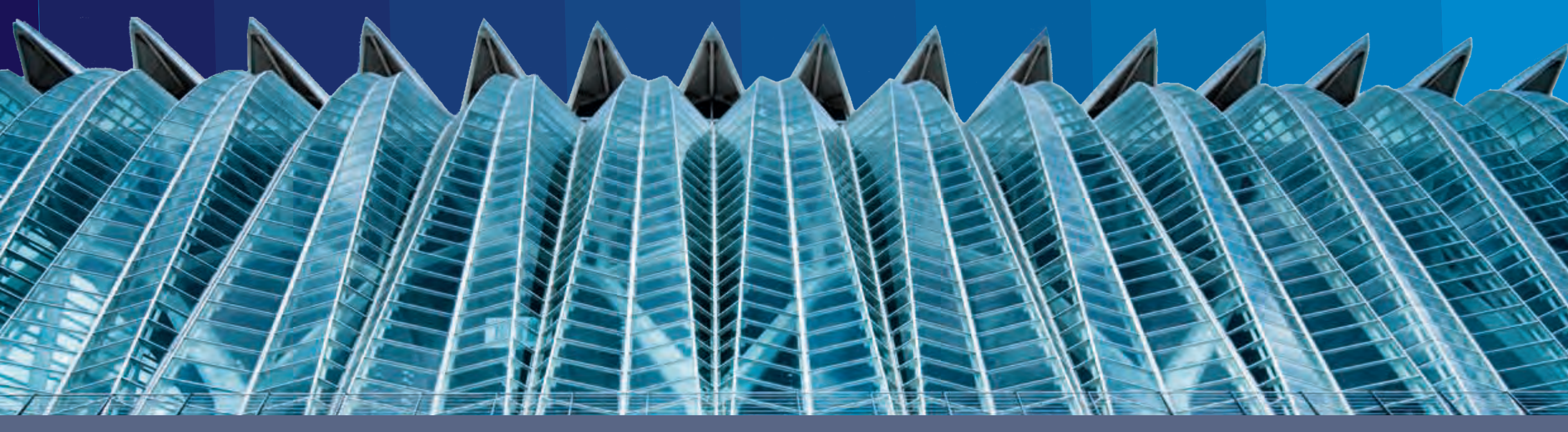
INTED 2018

12th International Technology, Education and Development Conference

5-7 March, 2018
Valencia (Spain)

CONFERENCE PROCEEDINGS



Rethinking Learning in a Connected Age

INTED 2018

12th **International Technology, Education** and **Development Conference**

5-7 March, 2018
Valencia (Spain)

CONFERENCE PROCEEDINGS

Published by
IATED Academy
iated.org

INTED2018 Proceedings
12th International Technology, Education and Development Conference
March 5th-7th, 2018 — Valencia, Spain

Edited by
L. Gómez Chova, A. López Martínez, I. Candel Torres
IATED Academy

ISBN: 978-84-697-9480-7
ISSN: 2340-1079
Depósito Legal: V-262-2018

Book cover designed by
J.L. Bernat

INTED2018 COMMITTEE AND ADVISORY BOARD

CONFERENCE SESSIONS

ORAL SESSIONS, 5th March 2018

Learning Analytics
Augmented Reality & 3D Videos
Experiential Learning
Ethical Issues & Digital Divide
Educational Leadership
Social Media in Education
Special Education Teaching
STEM Education Experiences

Teachers' Competencies Development and Assessment
Virtual Reality & Immersive Videos
Project-Based Learning in Higher Education
Models and strategies for facilitating work-integrated learning
ICT skills and competencies among Teachers (1)
Mobile Learning Apps
Employability Skills for the Special Needs Learner
Language Learning Education

e-Assessment
Learning Management Systems
Students Motivation & Engagement
Employability Issues and Challenges
ICT skills and competencies among Teachers (2)
Flipped Classroom
Multicultural Education Experiences
Computer-Assisted Language Learning

Assessment & e-Portfolios
Next Generation Classroom
Pedagogical and Didactical Innovations
Soft and Employability Skills
Teachers' Education
Flipping the STEM Classroom
Inclusive Education Experiences
Technology Enhanced Learning in Engineering Education

POSTER SESSIONS, 5th March 2018

Emerging Technologies in Education

Global Issues in Education and Research

ORAL SESSIONS, 6th March 2018

Research on Technology in Education
Technology Enhanced Learning
Service Learning & Community Engagement
Experiences in Business Education
Pre-service Teacher Education
International Cooperation Experiences
Maths Teaching and Learning (1)
Gender and Diversity Issues

Technological Issues in Education
Massive Open Online Courses & Open Educational Resources
Gamification & Game-based Learning
Lifelong Learning & Digital Skills
Teacher Training in the Digital Age
University-Industry Collaboration
Maths Teaching and Learning (2)
Language Learning: from EFL to EMI

Virtual Learning Environments
Sharing Digital Learning Content Internationally. Reality or wishful thinking?
Personalized Learning
Competence Assessment
Professional Development of Teachers
Entrepreneurship Education
Experiences in Early and Primary Education
Curriculum Design in Architecture & Civil Engineering

Virtual and Real Mobility Experiences
Media and Digital Literacy
Peer and Cooperative Learning
Links between Education and Research
Developing a Teacher Education Model for Preparing Teachers for the Future
Quality Assurance in Education
STEAM Education Experiences
New Experiences in Engineering Education

Active Learning Experiences
e-Learning & Distance Learning
Research on Education
Experiences in Health Sciencies Education
Educational Management
New Challenges of Higher Education Institutions
Science Popularization
21st Century Skills for Engineers

POSTER SESSIONS, 6th March 2018

Experiences in Education

Pedagogical Innovations and New Educational Trends

VIRTUAL SESSIONS

Apps for education
Barriers to Learning
Blended Learning
Collaborative and Problem-based Learning
Competence Evaluation
Computer Supported Collaborative Work
Curriculum Design and Innovation
Digital divide and acces to internet
Diversity issues and women and minorities in science and technology
E-content Management and Development
e-Learning
Education and Globalization
Education in a multicultural society
Educational Research Experiences
Educational Software and Serious Games
Enhancing learning and the undergraduate experience
Ethical issues in Education
Evaluation and Assessment of Student Learning
Experiences in STEM Education
Flipped Learning
ICT skills and competencies among teachers
Impact of Education on Development
Inclusive Learning
International Projects
Language Learning Innovations
Learning and Teaching Methodologies
Learning Experiences in Primary and Secondary School
Lifelong Learning
Links between Education and Research
Massive Open Online Courses (MOOC)
Mobile learning
New projects and innovations
New Trends in the Higher Education Area
Online/Virtual Laboratories
Organizational, legal and financial issues
Pedagogical & Didactical Innovations
Pre-service teacher experiences
Quality assurance in Education
Research Methodologies
Research on Technology in Education
Science popularization and public outreach activities
Student Support in Education
Technological Issues in Education
Technology-Enhanced Learning
Transferring disciplines
University-Industry Collaboration
Virtual Universities
Vocational Training

ABOUT INTED2018 Proceedings

HTML Interface: Navigating with the Web browser

This USB Flash drive includes all presented papers at INTED2018 conference. It has been formatted similarly to the conference Web site in order to keep a familiar environment and to provide access to the papers trough your default Web browser (open the file named "INTED2018_Proceedings.html").

An Author Index, a Session Index, and the Technical Program are included in HTML format to aid you in finding conference papers. Using these HTML files as a starting point, you can access other useful information related to the conference.

The links in the Session List jump to the corresponding location in the Technical Program. The links in the Technical Program and the Author Index open the selected paper in a new window. These links are located on the titles of the papers and the Technical Program or Author Index window remains open.

Full Text Search: Searching INTED2018 index file of cataloged PDFs

If you have Adobe Acrobat Reader version 6 or later (www.adobe.com), you can perform a full-text search for terms found in INTED2018 proceedings papers.

Important: To search the PDF index, you must open Acrobat as a stand-alone application, not within your web browser, i.e. you should open directly the file "INTED2018_FrontMatter.pdf" with your Adobe Acrobat or Acrobat Reader application.

This PDF file is attached to an Adobe PDF index that allows text search in all PDF papers by using the Acrobat search tool (not the same as the find tool). The full-text index is an alphabetized list of all the words used in the collection of conference papers. Searching an index is much faster than searching all the text in the documents.

To search the INTED2018 Proceedings index:

1. Open the Search PDF pane through the menu "Edit > Advanced Search" or click in the PDF bookmark titled "SEARCH PAPERS CONTENT".
2. The " INTED2018_index.pdx" should be the currently selected index in the Search window (if the index is not listed, click Add, locate the index file .pdx, and then click Open).
3. Type the search text, click Search button, and then proceed with your query.

For Acrobat 9 and later:

1. In the “Edit” menu, choose “Search”. You may receive a message from Acrobat asking if it is safe to load the Catalog Index. Click “Load”.
2. A new window will appear with search options. Enter your search terms and proceed with your search as usual.

For Acrobat 8:

1. Open the Search window, type the words you want to find, and then click Use Advanced Search Options (near the bottom of the window).
2. For Look In, choose Select Index.
3. In the Index Selection dialog box, select an index, if the one you want to search is available, or click Add and then locate and select the index to be searched, and click Open. Repeat as needed until all the indexes you want to search are selected.
4. Click OK to close the Index Selection dialog box, and then choose Currently Selected Indexes on the Look In pop-up menu.
5. Proceed with your search as usual, selecting other options you want to apply, and click Search.

For Acrobat 7 and earlier:

1. In the “Edit” menu, choose “Full Text Search”.
2. A new window will appear with search options. Enter your search terms and proceed with your search as usual.

A MULTI-PERSPECTIVE SIMULATOR FOR VISUALIZING AND ANALYZING THE KINEMATICS AND SINGULARITIES OF 2UPS/U PARALLEL MECHANISMS

A. Peidró, A. Belando, D. Valiente, O. Reinoso, L. Payá

Miguel Hernández University (SPAIN)

Abstract

This paper presents an educational simulation tool for graphically studying the forward kinematic problem and singularities of 2UPS/U parallel mechanisms. The presented tool allows the user to analyze the solutions of the forward kinematic problem from three perspectives: 1) as a set of eight discrete points in the complex domain, 2) as the zeroes of an eighth-degree characteristic polynomial, and 3) as the intersection points between two planar curves that represent the kinematic restrictions of the mechanism. Through several examples, we demonstrate the didactic usefulness of analyzing from these multiple perspectives the solutions of the forward kinematic problem and their relationships with singularities. We show that this is especially useful in degenerate situations, in which some perspectives are not valid since they fail to properly represent all the solutions of the forward kinematic problem.

Keywords: forward kinematics, parallel mechanism, simulator, singularities

1 INTRODUCTION

Robot manipulators are complex systems, so education in robotics requires using educational simulators [1] that allow the students to experiment with simulated robots and become familiar with some concepts before learning with real robots. This paper presents a simulation tool for analyzing the kinematics and singularities of parallel manipulators of type 2UPS/U. The presented simulator is part of the educational virtual laboratory PaRoLa (**Pa**rallel **Ro**botics **La**boratory, [2]), which is a web-based laboratory dedicated to the simulation of parallel robots and mechanisms. A parallel robot is a manipulator in which the motion of the gripper is controlled through two or more legs assembled and working in parallel. These robots have some advantages over the classical arm-like manipulators with open kinematic architecture found in industry, such as: higher stiffness, higher payload-to-weight ratio, and higher dynamic capabilities. However, parallel robots also have smaller ranges of operation (workspaces), their forward kinematic analysis is more complex, and they can reach singular configurations where it is not possible to completely control the motion of the gripper.

The 2UPS/U parallel mechanism, shown in Fig. 1, has a mobile body connected to a fixed body through a passive Universal joint (2UPS/**U**) and two parallel legs with Universal-Prismatic-Spherical architecture (2**UPS**/U), where the prismatic joint is actuated, i.e., we can control lengths d_1 and d_2 of the UPS legs by means of, for example, electric linear actuators. By controlling lenghts d_1 and d_2, it is possible to control the relative orientation between the fixed and mobile bodies. This relative orientation can be parameterized by angles (α, β), which are indicated in Fig. 1. This 2-degrees-of-freedom orientation mechanism has been studied by some authors [3,4].

In this paper, we present an educational simulator for studying the forward kinematic problem and singularities of this parallel mechanism. The main characteristic of the presented simulator is that it allows the user to study, visualize, and understand the solution of the forward kinematic problem from multiple complementary perspectives, which is especially useful when some of these perspectives fail to properly describe the solutions of this problem.

This paper is organized as follows. Section 2 presents and reviews the forward kinematic problem and singularities of 2UPS/U parallel mechanisms. Section 3 describes the educational simulation tool developed for interactively studying these two problems. Section 4 presents some examples of use of the presented educational tool. Finally, section 5 summarizes the main conclusions.

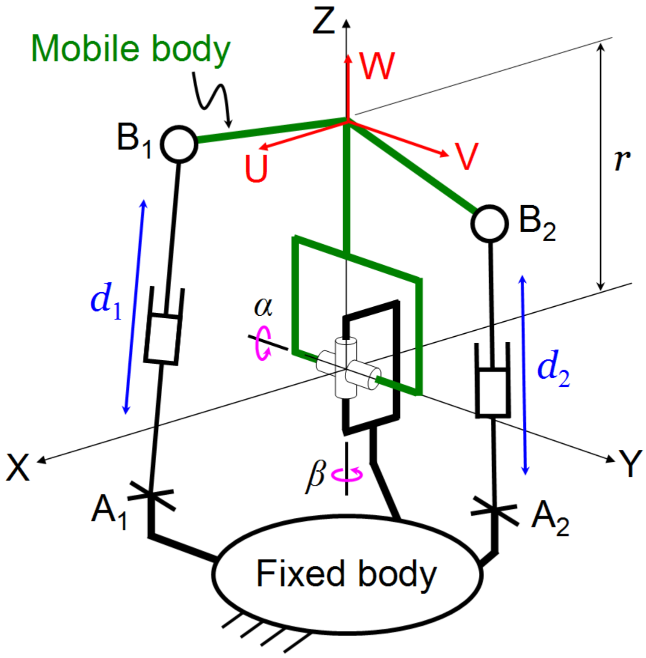


Figure 1. Schematic representation of the 2UPS/U parallel mechanism.

2 KINEMATICS AND SINGULARITIES OF 2UPS/U MECHANISMS

This section describes the two problems of 2UPS/U mechanisms that can be studied using the developed simulator: the forward kinematic problem, and the problem of singularities.

2.1 The forward kinematic problem of 2UPS/U mechanisms

The forward kinematic problem of this mechanism consists in determining the relative orientation between the fixed and mobile bodies in terms of lengths (d_1, d_2) of the UPS legs. The relative orientation between both bodies can be parameterized by angles (α, β), which are indicated in Fig. 1. In order to solve (α, β) in terms of (d_1, d_2), we must first write the kinematic restrictions of this mechanism, which are:

$$\left\| r\begin{bmatrix} \sin(\alpha)\cos(\beta) \\ \sin(\alpha)\sin(\beta) \\ \cos(\alpha) \end{bmatrix} + \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \mathbf{b}_1 - \mathbf{a}_1 \right\|^2 - d_1^2 = 0 \tag{1}$$

$$\left\| r\begin{bmatrix} \sin(\alpha)\cos(\beta) \\ \sin(\alpha)\sin(\beta) \\ \cos(\alpha) \end{bmatrix} + \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \mathbf{b}_2 - \mathbf{a}_2 \right\|^2 - d_2^2 = 0 \tag{2}$$

where r is the size of the mobile body, whereas $\mathbf{a}_i = \left[a_{ix}, a_{iy}, a_{iz}\right]^T$ $(i = 1{,}2)$ are the coordinates of the centers A$_i$ of the universal joints of the UPS legs, in frame XYZ (which is attached to the fixed body). Similarly, $\mathbf{b}_i = [b_{iu}, b_{iv}, b_{iw}]^T$ $(i = 1{,}2)$ are the coordinates of the centers B$_i$ of the spherical joints of the UPS legs, with respect to frame UVW (which is attached to the mobile body).

The forward kinematic problem consists, therefore, in solving α and β from Eqs. (1) and (2), for given known values of (d_1, d_2). This problem can be solved as follows:

1. Eqs. (1) and (2) involve trigonometric terms in angles α and β. Using the tangent half-angle substitution $(\sin x = (2 \cdot t_x)/(1 + t_x^2),\ \cos x = (1 - t_x^2)/(1 + t_x^2)$, with $t_x = \tan(x/2))$, these two equations can be transformed into two quadratic equations in t_α and t_β.
2. Using resultants, it is possible to eliminate t_β (or t_α) between these two quadratic equations, arriving at a single characteristic polynomial equation of degree 8 in t_α (or t_β, respectively):

$$k_8 t_\alpha^8 + k_7 t_\alpha^7 + k_6 t_\alpha^6 + k_5 t_\alpha^5 + k_4 t_\alpha^4 + k_3 t_\alpha^3 + k_2 t_\alpha^2 + k_1 t_\alpha + k_0 = 0 \quad (3)$$

where k_j denote (lengthy) coefficients that depend on (d_1, d_2), r, $\mathbf{a}_i$, and $\mathbf{b}_i$. Thus, the forward kinematic problem of the 2UPS/U mechanism has eight different solutions, i.e.: for a given pair (d_1, d_2), we will have eight different pairs (α, β) that simultaneously satisfy both Eqs. (1) and (2) (eight different relative orientations between the fixed and mobile bodies). In general, some of the solutions will be complex numbers (i.e., non-real, with non-zero imaginary part).

2.2 The problem of singularities in 2UPS/U parallel mechanisms

The singularities of the 2UPS/U mechanism are the configurations at which it is not possible to completely control the angular velocity of the mobile body by means of the controlled lengths (d_1, d_2). These configurations occur when $\det(\mathbf{J}) = 0$, where $\mathbf{J}$ is the Jacobian matrix:

$$\mathbf{J} = \begin{bmatrix} \partial e_1/\partial\alpha & \partial e_1/\partial\beta \\ \partial e_2/\partial\alpha & \partial e_2/\partial\beta \end{bmatrix} \quad (4)$$

and where e_1 and e_2 denote the left-hand sides of Eqs. (1) and (2), respectively. Equation $\{\det(\mathbf{J}) = 0\}$ defines the singularity locus of the mechanism, which is the geometric locus of all singular configurations of the 2UPS/U mechanism. This geometric locus can be represented in plane (d_1, d_2), which yields a set of singular curves. The representation of these singular curves in plane (d_1, d_2) is useful for the following purposes:

- For safely controlling the motion of the mechanism. When we control the 2UPS/U mechanism, actually we are controlling lengths (d_1, d_2) by means of, for example, electric linear actuators. Controlling (d_1, d_2) is equivalent to moving in plane (d_1, d_2). Therefore, it is important to know the shape and location of the singular curves in this plane, so that they can be avoided if we wish to completely control the angular velocity of the mobile body at all times.
- For studying transitions between different solutions of the forward kinematic problem. Parallel mechanisms can perform transitions between different solutions of the forward kinematics, by encircling cusp points of the singular curves in plane (d_1, d_2) [5].

The singular curves divide plane (d_1, d_2) into different regions, such that the number of real solutions of the forward kinematic problem is constant over each region (but the number of real solutions changes between adjacent regions).

Finally, it is important to remark that the concrete shape and location of the singular curves defined above depend on the geometric design parameters of the mechanism, which are r, $\mathbf{a}_i$, and $\mathbf{b}_i$. When modifying these parameters, the singular curves transform as a consequence, which may result in wider (or narrower) singularity-free regions in plane (d_1, d_2), or in a higher (or smaller) presence of cusps in these curves, etc. This can be easily visualized and analyzed using the developed educational simulator, which is presented next.

3 DEVELOPED EDUCATIONAL SIMULATOR OF 2UPS/U PARALLEL MECHANISMS (USER'S MANUAL)

This section presents and describes the functionalities of the educational simulator developed for studying and visualizing the forward kinematic problem and the singularities of 2UPS/U parallel mechanisms.

The developed simulator is available and can be downloaded from the website of PaRoLa, the educational virtual laboratory of parallel robots: http://arvc.umh.es/parola/2ups_u.html. After downloading it, the latest version of Java may be needed in order to properly run it.

This simulator has been developed using Easy Java Simulations (EJS) [6]. EJS is an intuitive authoring tool for developing educational simulators in Java and Javascript programming languages. EJS greatly facilitates the task of programming educational graphical user interfaces, so that the designer of the simulator only needs to focus on the educational contents of the simulator, without worrying about low-level implementation details.

The developed simulator is shown in Fig. 2. This simulator has three windows: w1, w2, and w3. Let us begin by analyzing window w1, which has three panels p1, p2, and p3:

- Panel p1 represents a schematic three-dimensional view of the 2UPS/U mechanism.
- Panel p2 represents plane (d_1, d_2), with singular curves represented in blue. This panel also represents a tiny magenta square, whose coordinates are the current values of (d_1, d_2).
- Panel p3 is a control panel in which the user can use several sliders and numeric fields to modify d_1, d_2, or any of the geometric design parameters of the mechanism (r, $\mathbf{a}_i$, $\mathbf{b}_i$). When the user modifies any of these geometric design parameters, the singular curves of panel p2 are deformed accordingly.

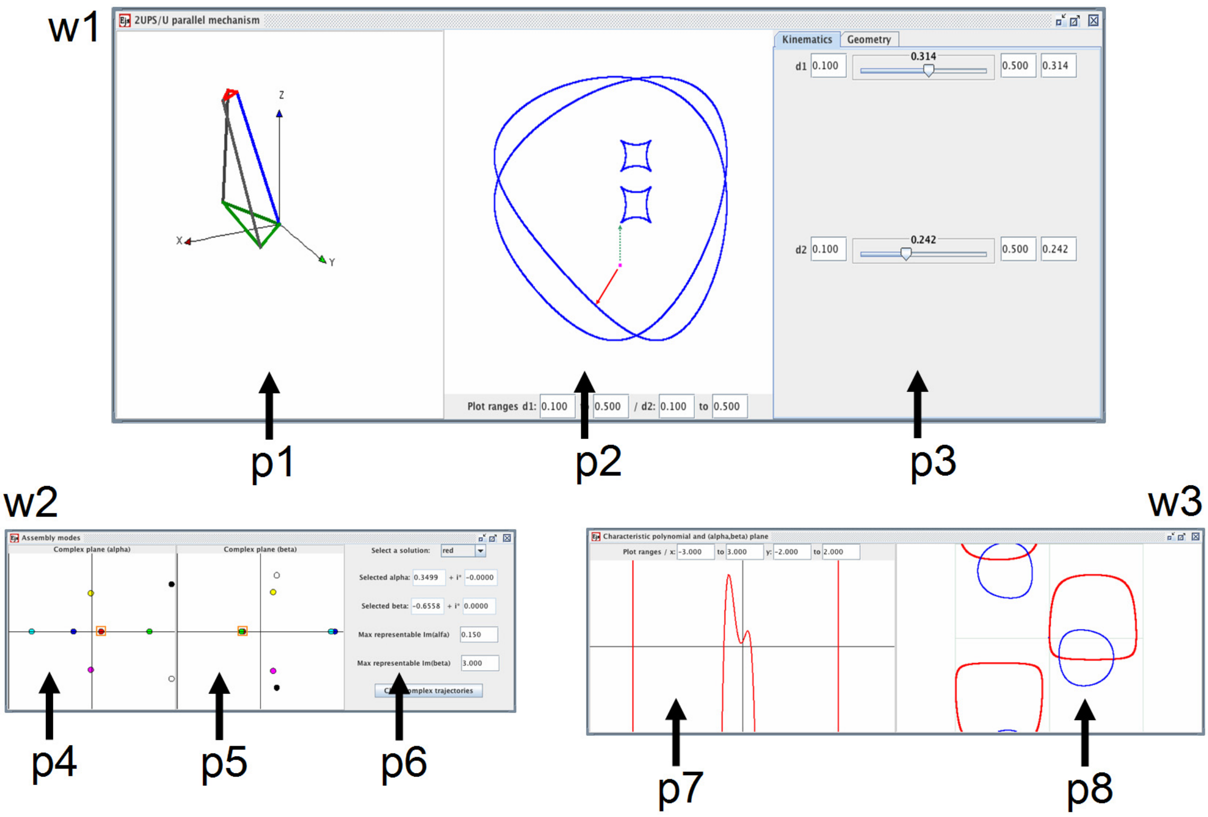


Figure 2. Windows and panels of the developed simulator.

Window w1 allows the user to simulate the forward kinematic problem. Recall from section 2.1 that the forward kinematic problem consists in inserting known values of (d_1, d_2) and computing the resulting angles (α, β). To insert or specify lengths (d_1, d_2), the user can use the sliders and numeric fields of control panel p3. Alternatively, it is possible to click and drag the tiny magenta square in panel p2, which allows the user to simultaneously vary both d_1 and d_2. When varying (d_1, d_2) by any of these means, the simulator automatically solves the forward kinematic problem, obtanining angles (α, β), and it represents the corresponding posture of the mechanism in panel p1. Although panel p1 only represents one of the possible solutions of the forward kinematic problem, recall from section 2.1 that this problem has eight different solutions. Where are the remaining seven solutions represented?

The forward kinematic problem of the 2UPS/U parallel mechanism has eight different solutions since its characteristic polynomial (3) has degree 8. In general, some of these solutions will be complex, i.e., α and β will be complex numbers with non-zero imaginary part (complex solutions are not physically possible, i.e., for these solutions the mechanism cannot be assembled and would remain mechanically "opened"). The number of complex solutions obtained after solving the forward kinematic problem will depend on the precise values of (d_1, d_2) and (r, $\mathbf{a}_i$, $\mathbf{b}_i$) (for example, it is easy to see in Fig. 1 that, if d_1 and d_2 are much larger than the remaining dimensions of the mechanism, then it will be impossible to assemble the mechanism and all eight solutions will be non-real). The eight solutions of the forward kinematic problem are represented in window w2 of the developed simulator, which also has three panels p4, p5, and p6:

- Panel p4 represents the eight solutions for angle α, in the complex plane. Each solution is represented in a different color: red, green, blue, cyan, magenta, yellow, black, and white.
- Panel p5 represents the eight solutions for angle β in the complex plane, following the same convention for colors as in panel p4. This means that, for example, red solutions in panels p4 and p5 form a solution pair (α, β) that simultaneously satisifes Eqs. (1) and (2) (and so on for the remaining colors).
- Panel p6 is another control panel, in which the user can choose which solution (out of the eight solutions) should be represented in panel p1. Alternatively, the user can also select which solution to represent by directly clicking the desired solution in panels p4 or p5 (the selected solution appears enclosed by an orange square in both complex planes). Panel p6 also allows the user to vary the representation ranges for the imaginary axes of panels p4 and p5. The range of representation for the real axes of these panels is always $[-\pi, \pi]$ radians. This is because the complex solutions of (α, β) always have real parts which are angles [7] and, therefore, always wrap to interval $[-\pi, \pi]$ (angles differing by an integer multiple of 2π radians can be considered equivalent, for our purposes). This means that a solution leaving panel p4 (or p5) through its right edge (real part equal to π), will reappear on its left edge (real part equal to $-\pi$), and vice versa.

Note that, if the user selects a complex solution, the represented posture in panel p1 will have no physical meaning, since this posture is computed using only the real parts of α and β.

Finally, the simulator has a third window w3, which has two panels p7 and p8. These two panels provide alternative perspectives of the forward kinematic problem:

- Panel p7 represents the graph of the characteristic polynomial of the left-hand side of Eq. (3). In this case, the solutions of the forward kinematic problem are the zeroes of the polynomial, i.e., the points where this graph intersects the horizontal axis.
- Panel p8 represents (real) plane (α, β) for $-\pi \leq \alpha \leq \pi$ and $-\pi \leq \beta \leq \pi$. Two curves are represented in this plane, which are the graphical representations of Eqs. (1) (red curve) and (2) (blue curve). Note that, for given geometric design parameters (r, $\mathbf{a}_i$, $\mathbf{b}_i$) and lengths (d_1, d_2), each of these two equations defines a curve in plane (α, β). According to this representation, the real solutions of the forward kinematic problem are given by the intersection points of the red and blue curves of panel p8, since these intersection points simultaneously satisfy both Eqs. (1) and (2).

To sum up: the developed simulator allows the user to analyze and visualize the forward kinematic problem from three complementary perspectives:

- As a set of eight solutions in complex planes of variables α and β.
- As the intersection between polynomial (3) and the horizontal axis.
- As the intersection of planar curves (1) and (2) in real plane (α, β).

In these three cases, the solutions of the forward kinematic problem vary as the user varies d_1 or d_2: the complex solutions of panels p4 and p5 describe trajectories along their respective complex planes, the characteristic polynomial (3) deforms and its zeroes vary accordingly, and the curves (1) and (2) in real plane (α, β) also deform, with their intersection points varying as a consequence.

Note that, unlike the representations of panels p4 and p5, the representations of panels p7 and p8 only allow the user to visualize the real solutions of the forward kinematic problem (i.e., real intersection points between planar curves). However, visualizing the same problem (the forward kinematic problem) from all these perspectives is very useful to better understand it, especially in singular degenerate cases in which some of these representations are not valid and miss important information. This will be illustrated in next section through some examples.

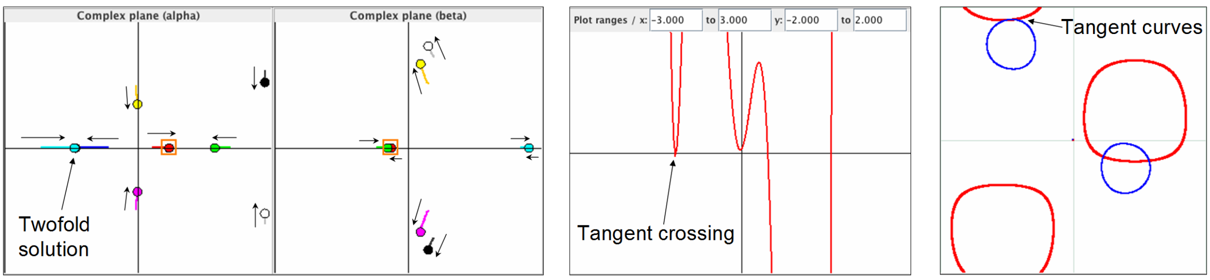
4 ILLUSTRATIVE EXAMPLES OF THE USE OF THE DEVELOPED EDUCATIONAL SIMULATOR

This section presents some examples that demonstrate the usefulness of the developed educational simulator to analyze the singularities and solutions of the forward kinematic problem of 2UPS/U parallel mechanisms from the different perspectives presented in section 3.

4.1 Examples of simple, twofold, and threefold solutions

For this example, we will use the default geometry of the simulator, which is: $r = 0.3$, $\mathbf{a}_1 = [0.1, 0.1, 0]^T$, $\mathbf{a}_2 = [0.1, -0.1, 0.03]^T$, $\mathbf{b}_1 = [0.03, 0.03, 0]^T$, $\mathbf{b}_2 = [0.03, -0.03, 0]^T$. This corresponds to a fairly "regular" 2UPS/U mechanism, as shown in panel p1 of Fig. 2. The singular curves in plane (d_1, d_2) for this geometry are shown in panel p2 of Fig. 2. Let us analyze next the solutions of the forward kinematic problem for $(d_1, d_2) = (0.314, 0.242)$, which is the position of the tiny magenta square in panel p2 of Fig. 2. This position is a regular configuration, at which the forward kinematic problem has eight different solutions, each with multiplicity 1 (i.e., eight simple solutions). This can be seen in the complex planes of α and β in panels p4 and p5 of Fig. 2: the eight complex solutions are different. As these panels show, in this example four solutions are real and four are complex. The four real solutions can also be visualized in panels p7 and p8 of Fig. 2: in panel p7, the graph of the characteristic polynomial (3) intersects the horizontal axis four times. Similarly, panel p8 of Fig. 2 shows the four intersections between curves (1) and (2), with all four intersections being simple or secant.

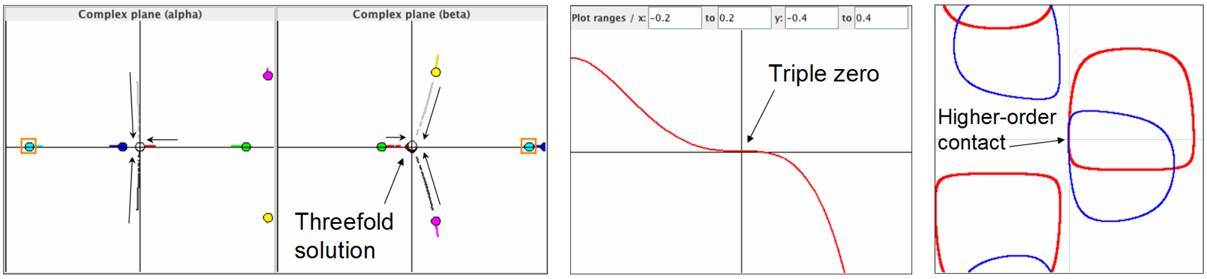
Next, let us analyze what happens when describing the diagonal red trajectory shown in panel p2 of Fig. 2, which begins at the magenta square with coordinates $(0.314, 0.242)$, and ends at singularity $(d_1, d_2) \approx (0.284, 0.198)$. When lengths (d_1, d_2) describe this trajectory toward the singularity, the solutions (α, β) describe the trajectories shown in Fig. 3a, in the respective complex planes. As this figure shows, blue and cyan solutions (which are different at the beginning of the trajectory) coalesce, becoming a twofold solution. This is what happens when approaching singularities: at least two different solutions coalesce (more than two solutions coalesce when the approached singularity is "more special"). This twofold solution can also be visualized in the graph of the characteristic polynomial (3), since this case corresponds to a tangent crossing between this graph and the horizontal axis (Fig. 3b). Finally, this twofold solution is also visualized in the real plane (α, β), where curves (1) and (2) become tangent (see Fig. 3c).



(a) (b) (c)

Figure 3. (a) Evolution of complex solutions when approaching an ordinary singularity. (b) Double zero of the characteristic polynomial. (c) Tangent contact between curves defined by Eqs. (1) and (2).

Finally, let us analyze the solutions of the forward kinematic problem when lengths (d_1, d_2) describe the vertical dotted green trajectory indicated in panel p2 of Fig. 2. This trajectory begins again at magenta square with coordinates $(d_1, d_2) = (0.314, 0.242)$, but it ends now at the cuspidal singularity with coordinates $(d_1, d_2) \approx (0.315911, 0.287575)$. When the lengths (d_1, d_2) describe this trajectory, we see in Fig. 4a that the red, black, and white solutions coalesce in both complex planes of α and β. This means that we have a threefold solution (threefold solutions occur at cusp singularities). This triple solution is also reflected in the graph of polynomial (3), which intersects the horizontal axis at a triple zero (Fig. 4b). Finally, this threefold solution can also be visualized in real plane (α, β), where curves (1) and (2) have a higher-order contact near the origin (Fig. 4c).

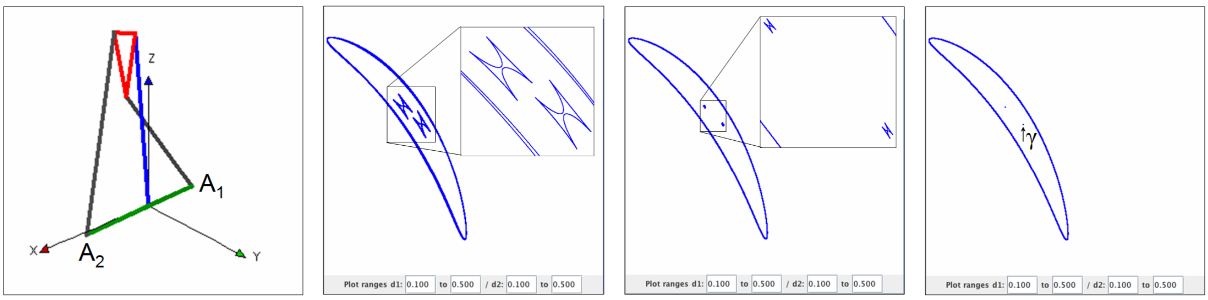


(a) (b) (c)

Figure 4. (a) Coalescence of three solutions when approaching a cusp in plane (d_1,d_2). (b) Triple zero of the characteristic polynomial. (c) Higher-order contact between curves defined by Eqs. (1) and (2).

4.2 Example of a degenerate solution

Consider now a 2UPS/U mechanism with the following geometry: $r = 0.3$, $\mathbf{a}_1 = [-0.1,0,0]^T$, $\mathbf{a}_2 = [0.15,0.01,0]^T$, $\mathbf{b}_1 = [-0.03,0.03,-0.1]^T$, $\mathbf{b}_2 = [0.03,0.03,0]^T$. As Fig. 5a shows, this mechanism has universal joint A_1 lying exactly on axis X ($a_{1y} = 0$), whereas universal joint A_2 is *almost* lying on the same axis ($a_{2y} = 0.01$, which is small but not zero). Fig. 5b shows the singular curves of this mechanism in plane (d_1, d_2). As the zoomed area of this figure shows, the singular curves have two "diamonds", which are closed curves having four cusps [8] (note that the singular curves of panel p2 of Fig. 2 also exhibit two diamonds of this type). If a_{2y} is further reduced to 0.001, such that universal joint A_2 gets even closer to axis X, these two diamonds become smaller (see Fig. 5c). In the limit, when A_2 lies exactly on axis X (like joint A_1), the two aforementioned diamonds degenerate and become isolated points (Fig. 5d). Isolated singularities like these have interesting effects on parallel mechanisms [9]. Therefore, we can use the developed simulator to study what happens near the isolated singularities of Fig. 5d.



(a) (b) (c) (d)

Figure 5. (a) A 2UPS/U mechanism with almost aligned fixed body. As universal joint A_2 gets closer to axis X, the diamonds of (b) become smaller (c) and eventually degenerate into points (d).

For example, following the same procedure as in previous examples, let us use the simulator to check how the solutions of the forward kinemaic problem evolve as lengths (d_1, d_2) tend to the rightmost isolated singularity of Fig. 5d, whose coordinates are: $(d_1, d_2) \approx (0.24042, 0.3245)$. For example, we can set $d_1 = 0.24042$ in the simulator and vary d_2 from 0.3 to 0.3245 using the sliders. This would approach this isolated singularity from below, through vertical trajectory γ indicated in Fig. 5d. By doing this, the solutions evolve in the complex plane of β as illustrated in Fig. 6a, in which six different solutions (two real and four complex – note that two pairs of complex solutions are coincident) seem to converge at $\beta = -\pi/2$ rad. Therefore, one may think that we have a sixfold solution at the isolated singularity, besides two simple solutions indicated in Fig. 6a. However, checking the evolution of these solutions in the complex plane of α reveals that these six solutions do not converge at all for the α coordinate, which seems a strange behavior (one would expect both α and β coordinates of these solutions to coalesce simultaneously).

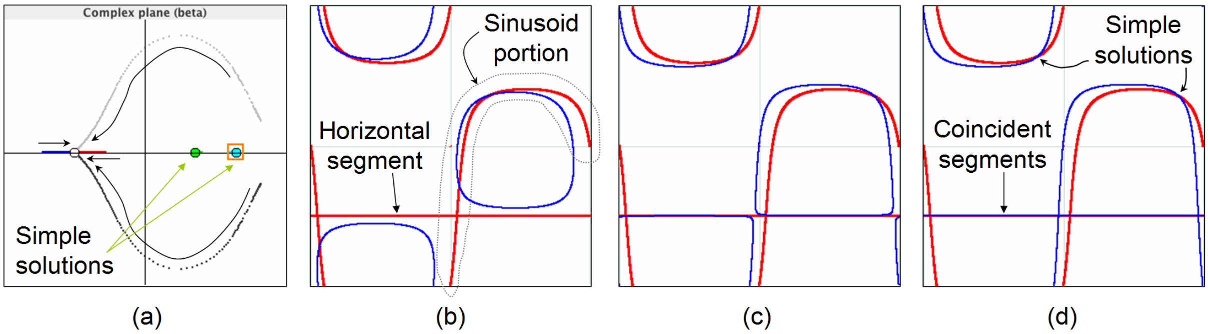


Figure 6. (a) Six solutions seem to coalesce in the complex plane of β when approaching the rightmost isolated singularity of Fig. 5d (magenta and yellow complex solutions coincide with white and black complex solutions, respectively, and are therefore overlapped in Fig. 6a). (b) Curves defined by Eqs. (1) and (2) at the beginning of the trajectory (d_2 = 0.3). (c) The same curves for d_2 = 0.324. (d) The same curves for d_2 = 0.3245 (isolated singularity approached).

To understand what is actually going on, it is necessary to analyze the forward kinematic problem as the intersection of curves (1) and (2) in real plane (α, β), in panel p8 of the simulator. Figs. 6b to 6d show the evolution of these curves as lengths (d_1, d_2) approach the studied isolated singularity through vertical trajectory γ. Since this trajectory is vertical, d_1 is kept constant and red curve (1), which only depends on d_1, does not change along the trajectory. This red curve consists of two intersecting portions: a horizontal segment $\beta = -\pi/2$ and an (approximately) sinusoid portion.

On the other hand, the blue curves (2), which only depend on d_2, are two loops. During the execution of trajectory γ, the red and blue curves intersect at four different points (and all four intersections are simple). However, as Figs. 6b-6d show, the blue curves deform and adopt a similar shape to the red curves as the isolated singularity is approached in plane (d_1, d_2). In the limit, when the isolated singularity is approached, the blue curves consist also of two intersecting portions, like the red curves: a horizontal segment $\beta = -\pi/2$ and another (approximately) sinusoid portion. And this is where the previous apparently strange behavior of the complex solutions can be understood, as explained next.

The sinusoid portions of the red and blue curves of Fig. 6d intersect at two points (and these intersections are simple). These are the simple solutions indicated in Fig. 6a. However, the horizontal portions of these curves are coincident, i.e., these portions have infinitely many different intersection points. This means that, in addition to the two mentioned simple solutions, this degenerate mechanism admits infinitely many solutions with $\beta = -\pi/2$, with angle α being free: the mechanism can freely rotate about axis X, without control (visualize this in Fig. 1 for $\beta = -\pi/2$ and joints A_1 and A_2 lying on axis X). This is a special self-motion singularity, in which the linear actuators are locked (lengths d_1 and d_2 are constant) but the mechanism admits finite uncontrolled motions.

This example demonstrates the importance of analyzing the forward kinematic problem from different perspectives, as allowed by the developed tool. Although visualizing the solutions in the complex planes is useful to understand how different real and complex solutions coalesce at singularities, this representation is only valid when the forward kinematic problem admits a finite number of discrete solutions. In degenerate cases like the one studied in this subsection, it is necessary to visualize the forward kinematic problem from a more global perspective, such as the intersection of the planar curves that represent the kinematic restrictions of the mechanism. In these cases, representations of discrete solutions miss important information and solutions, as we have seen in Fig. 6a.

5 CONCLUSIONS

In this paper, we have presented an intuitive graphical and educational simulator, which allows the students to easily analyze the forward kinematics and singularities of 2UPS/U parallel mechanisms from multiple complementary perspectives. Through several examples of use of the presented tool, we have demonstrated the usefulness of these multiple perspectives for analyzing and understanding the solutions of the forward kinematic problem, as well as their relationships to singularities. The visualization of the forward kinematic problem as the problem of intersecting two planar curves, which represent the kinematic constraints of the mechanism, has proven to be especially useful and interesting to properly understand the solution of the forward kinematics in degenerate situations.

ACKNOWLEDGEMENTS

This work was supported by the Spanish Ministries of Education (grant No. FPU13/00413) and Economy (project No. DPI 2016-78361-R).

REFERENCES

[1] A. Gil, Ó. Reinoso, J. M. Marín, L. Payá, and J. Ruiz, “Development and deployment of a new robotics toolbox for education,” *Comput Appl Eng Educ*, vol. 23, no. 3, pp. 443–454, 2015.

[2] A. Peidró, A. Gil, J. M. Marín, and Ó. Reinoso, “A Web-based Tool to Analyze the Kinematics and Singularities of Parallel Robots,” *Journal of Intelligent & Robotic Systems*, vol. 81, no. 1, pp. 145–163, 2016.

[3] M. Carricato and V. Parenti-Castelli, “A Novel Fully Decoupled Two-Degrees-of-Freedom Parallel Wrist,” *International Journal of Robotics Research*, vol. 23, no. 6, pp. 661–667, 2004.

[4] Y. Xu, L. Chen, W. Yan, J. Yao, and Y. Zhao, “Motion Decoupling Analysis of a Kind of 2R Parallel Mechanism with Two Continuous Rotational Axes” in *Advances in Reconfigurable Mechanisms and Robots II* (X. Ding, X. Kong, and J.S. Dai eds.), pp. 307–313, Switzerland: Springer International Publishing, 2016.

[5] M. Zein, P. Wenger, and D. Chablat, “Non-singular assembly-mode changing motions for 3-RPR parallel manipulators,” *Mech. Mach. Theory*, vol. 43, no. 4, pp. 480–490, 2008.

[6] F. Esquembre, “Easy Java Simulations: a software tool to create scientific simulations in Java,” *Computer Physics Communications*, vol. 156, no. 2, pp. 199–204, 2004.

[7] A. Peidró, Ó. Reinoso, A. Gil, J. M. Marín, and L. Payá, “A Simulation Tool to Study the Kinematics and Control of 2RPR-PR Parallel Robots,” *IFAC-PapersOnLine*, vol. 49, no. 6, pp. 268–273, 2016.

[8] M. Coste, D. Chablat, and P. Wenger, “Perturbation of Symmetric 3-RPR Manipulators and Asymptotic Singularities” in *New Trends in Mechanism and Machine Science* (F. Viadero and M. Ceccarelli eds.), pp. 23–31, Dordrecht: Springer Science+Business Media, 2013.

[9] A. Peidró, Ó. Reinoso, A. Gil, J. M. Marín, L. Payá, and Y. Berenguer, “Second-order taylor stability analysis of isolated kinematic singularities of closed-chain mechanisms,” in *Proceedings of the 14th International Conference on Informatics in Control, Automation, and Robotics (ICINCO 2017)*, vol. 2, pp. 351–358, 2017.